Averages of impact factors

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It is shown that the ratio of the harmonic mean of citations over the harmonic mean of publications does not lead to an acceptable impact measure for a meta-journal. This result contrasts markedly with the corresponding cases in which the arithmetic or geometric average is used. The relation between different averages and the regression of impact over publications or the regression of the opposite of impact over the opposite of publication numbers is studied in some detail. This leads to the general observation that if the regression line of y over x has a positive slope then this is not necessarily true for the regression line of 1/y over 1/x.

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Impact

Consider a group of journals and assume that for each of these journals an impact factor has been determined. In this note it does not matter which type of impact factor this is: the classical Garfield-Sher impact factor, another synchronous impact factor, a diachronous impact factor or even a more general one (Ingwersen *et al.*, 2001; Frandsen & Rousseau, 2005). In any case a journal impact factor is a quotient of the type 'number of citations' divided by 'corresponding number of publications'. Formally, we denote the impact factor of journal j as:

$$I_j = \frac{C_j}{P_j}$$

This is: the impact factor of journal j, denoted as I_j, is equal to the number of citations received by journal j (C_j) over a given time period divided by the corresponding number of articles published in journal j (P_j). Of course, we will always assume that $P_j \neq 0$.

Assume now that a group of journals is considered as one whole and that one wants to determine an impact factor for this ensemble. As in earlier publications (Egghe & Rousseau, 1996a, b) we will refer to this larger group as a meta-journal. Note that, mutatis mutandis, what will be said here for journal impact factors may also be applied to web impact factors (Ingwersen, 1998) and similar ratios.

Averages of journal impacts

Assume that a meta-journal contains n journals, each with an impact factor denoted as I_j, j = 1, ..., n. An obvious way to determine an impact factor for this meta-journal is taking the arithmetic average of all impacts:

$$\frac{1}{n}\sum_{j=1}^{n} I_j = \frac{1}{n}\sum_{j=1}^{n} \frac{C_j}{P_j} \quad (1)$$

This impact factor is called the average impact factor (AIF) in (Egghe & Rousseau, 1996a). Besides the AIF we also introduced a Global Impact Factor (GIF):

$$GIF = GIF_A = \frac{\frac{\sum_{j=1}^{n} C_j}{n}}{\frac{\sum_{k=1}^{n} P_k}{n}} = \frac{\frac{\sum_{j=1}^{n} C_j}{n}}{\frac{\sum_{k=1}^{n} P_k}{n}} = \frac{\mu_C}{\mu_P} \quad (2)$$

where μ_C and μ_P denote the average number of citations and the average number of published articles of the journals constituting the meta-journal. As this global impact factor is constructed using arithmetic averages, we denoted it as GIF_A. Similarly, from now on, we denote AIF as AIF_A. We observe that the GIF_A can be rewritten as:

$$GIF_A = \frac{\frac{\sum_{j=1}^{n} C_j}{n}}{\sum_{k=1}^{n} P_k} = \sum_{j=1}^{n} \frac{\frac{P_j}{n}\frac{C_j}{P_j}}{\sum_{k=1}^{n} P_k} = \sum_{j=1}^{n} \frac{\frac{P_j}{n} I_j}{\sum_{k=1}^{n} P_k} \quad (3)$$

Equation (3) shows that the GIF_A is actually a "weighted" average of the individual impact factors I_j. Strictly speaking one could argue that (3) is not a weighted average as the variables that are weighted depend on the weights (here the I_j are functions of the P_j). Anyway, equation (3) makes it clear why the GIF_A is a good impact factor for a meta-journal: journals that publish relatively more articles have a larger contribution to the GIF_A than journals that publish a relatively lower number of articles. We note that if all P_j are equal then $AIF_A = GIF_A$.

From a ratio of arithmetic means to a ratio of geometric ones

Equation (2) leads to the idea of replacing the arithmetic mean by another kind, such as the geometric mean (denoted as G). The resulting impact factor is then denoted as GIF_G. It is easily seen that the ratio of the geometric mean of citations over publications is equal to the geometric mean of the impacts.

$$GIF_G = \frac{G_C}{G_P} = \frac{\sqrt[n]{C_1.C_2...C_n}}{\sqrt[n]{P_1.P_2...P_n}} = \sqrt[n]{\frac{C_1}{P_1}\frac{C_2}{P_2}\cdots\frac{C_n}{P_n}} = \sqrt[n]{I_1 I_2 \ldots I_n} = G_1 = AIF_G \quad (4)$$

Here AIF_G is just another notation for the geometric average of the impact factors of the journals making up the meta-journal.

What about a ratio of harmonic means as another global impact measure?

What happens if we replace the arithmetic mean by the harmonic mean? Is this a good idea? Replacing the arithmetic mean in (2) by the harmonic mean, denoted as HM, yields:

$$GIF_H = \frac{HM_C}{HM_P} = \frac{\frac{n}{\sum_{k=1}^{n} \frac{1}{C_k}}}{\frac{n}{\sum_{j=1}^{n} \frac{1}{P_j}}} = \frac{\sum_{j=1}^{n} \frac{1}{P_j}}{\sum_{k=1}^{n} \frac{1}{C_k}} \quad (5)$$

Note that we have to assume now that all $C_k \neq 0$. The harmonic global impact (GIF_H) clearly increases if one C_j increases; similarly, it decreases if one P_j increases. These are good properties. Yet, GIF_H is not an acceptable measure for the impact of a meta-journal. Indeed, consider the following calculation:

$$GIF_H = \frac{\sum_{j=1}^{n} \frac{1}{P_j}}{\sum_{k=1}^{n} \frac{1}{C_k}} = \sum_{j=1}^{n} \frac{1}{\sum_{k=1}^{n} \frac{1}{C_k}} \frac{1}{P_j} = \sum_{j=1}^{n} \frac{1}{\sum_{k=1}^{n} \frac{C_j}{C_k}} \frac{C_j}{P_j} = \sum_{j=1}^{n} \frac{1}{\sum_{k=1}^{n} \frac{C_j}{C_k}} I_j \quad (6)$$

Equation (6) shows that also this expression can be written as a "weighted" mean of impacts. Yet, the weights are proportional to the opposite of the relative number of citations, namely.

$$\left(\frac{1}{\sum_{k=1}^{n} \frac{C_j}{C_k}} \right)$$

This is certainly NOT what one expects from an acceptable meta impact factor. We conclude that taking a ratio of harmonic means as an impact factor is not a good idea. This is illustrated in the following example.

An example

Consider the following three cases (Table 1). The number of publications for each journal is denoted as P (with superscripts 1, 2 and 3 for the three cases), and the number of citations is denoted as C (with corresponding superscripts 1, 2 and 3). In each case the impact factors of the individual journals stay invariant.

Table 1 An example

Journals	P^1	C^1	P^2	C^2	P^3	C^3
J_1	10	10	100	100	10	10
J_2	10	20	10	20	10	20
J_3	10	30	10	30	10	30
J_4	10	40	10	40	100	400

The AIF is equal to 2.5 in the three cases. The GIF_As are: $GIF_A^1 = 2.5$, $GIF_A^2 = 19/13 = 1.46$ and $GIF_A^3 = 46/13 = 3.54$. In our opinion these GIF_As reflect better the impact of the four journals considered as one (the meta-journal) than the AIFs. Finally, the GIF_Hs are: $GIF_H^1 = 1.92$; $GIF_H^2 = 2.62$ and $GIF_H^3 = 1.67$. As predicted the GIF_H change in the opposite direction of what one naturally expects.

Further discussions about the use of the harmonic mean as an impact measure

As the harmonic global impact (GIF_H) satisfies the basic properties of an impact factor, namely increasing if one C_j increases and decreasing if one P_j increases, we want to look somewhat deeper into properties of GIF_H, the harmonic global impact.

First, we observe that there exists another possibly acceptable impact factor for a meta-journal which has not been mentioned yet, namely the harmonic means of the impact factors of the journals included in the meta-journal. As this is another type of average impact factor it is denoted as AIF_H.

$$AIF_H = \frac{n}{\sum_{j=1}^{n} \frac{1}{I_j}} = \frac{n}{\sum_{j=1}^{n} \frac{P_j}{C_j}} \tag{7}$$

Recall that $1/I_j$ has been termed the indifference factor of journal j (denoted as D_j) in (Egghe & Rousseau, 1996a). Hence, AIF_H is equal to one over the arithmetic average of the indifference factors of each journal. Considering again Table 1 we see that $AIF_H{}^1 = 1.92 = AIF_H{}^2 = AIF_H{}^3$. Note that if all P_j are equal, $AIF_H = GIF_H$.

If $\rho(X, Y)$ denotes the slope of the regression line of Y (ordinate) over X (abscissa), then the following theorem holds.

Theorem 1 (Egghe & Rousseau, 1996a)

$$GIF_A > AIF_A$$
$$\Leftrightarrow$$
$$\rho(P, I) > 0$$

Similar results hold when $GIF_A = AIF_A$ or $GIF_A < AIF_A$.
A somewhat similar result can be shown regarding the harmonic means.

Theorem 2

$$GIF_H > AIF_H$$
$$\Leftrightarrow$$
$$\rho\left(\frac{1}{P}, \frac{1}{I}\right) < 0$$

And similar results hold for $GIF_H = AIF_H$ and $GIF_H < AIF_H$.

Proof.

$$GIF_H > AIF_H$$
$$\Leftrightarrow$$
$$\frac{1}{GIF_H} < \frac{1}{AIF_H}$$
$$\Leftrightarrow$$

$$\frac{\sum_{j=1}^{n}\frac{1}{C_j}}{\sum_{j=1}^{n}\frac{1}{P_j}} < \frac{1}{n}\sum_{j=1}^{n}\frac{P_j}{C_j}$$

$$\Leftrightarrow$$

$$\rho\left(\frac{1}{P}, \frac{1}{I}\right) < 0$$

The last equivalence follows from theorem 1 with the variable 1/P in the role of P and the variable 1/C in the role of C.

Definition (Egghe & Rousseau, 2002)

A scatter plot $\{(x_j,y_j), j = 1, \ldots, n\}$ is said to be increasing if $x_i < x_j$ if and only if $y_i < y_j$. Similarly we define a decreasing scatter plot.
A scatter plot $\{(x_j,y_j), j = 1, \ldots, n\}$ is said to be decreasing if $x_i < x_j$ if and only if $y_i > y_j$.
A scatter plot $\{(x_j,y_j), j = 1, \ldots, n\}$ is said to be constant for all i, j: $y_i = y_j$.

If a scatter plot is either increasing, decreasing or constant, then it is said to be monotone.

Using this definition we formulate the following theorem.

Theorem 3

Assume that the scatter plot $\{(P_j, I_j), j = 1, \ldots, n\}$ is increasing, then $GIF_A > AIF_A$ and $GIF_H < AIF_H$.

Assume that the scatter plot $\{(P_j, I_j), j = 1, \ldots, n\}$ is decreasing, then $GIF_A < AIF_A$ and $GIF_H > AIF_H$.

Proof. Assume that the scatter plot $\{(Pj, Ij), j = 1, \ldots, n\}$ is increasing then $\rho(P, I) > 0$ by Corollary 1 in (Egghe & Rousseau, 1996a). Hence by Theorem 1 $GIF_A > AIF_A$. (This result is also mentioned in (Egghe & Rousseau, 2002)).

If the scatter plot $\{ (Pj, Ij), j = 1, \ldots,n\}$ is increasing, then, clearly, the scatter plot $\{ (Pj, 1/Ij), j = 1, \ldots,n\}$ is decreasing, and the scatter plot $\{ (1/Pj, 1/Ij), j = 1, \ldots,n\}$ is increasing hence $\rho(1/P,1/I) > 0$. It follows by Theorem 2 that $GIF_H < AIF_H$. The other statements of Theorem 3 can be shown in a similar way.

Remark. If all I_j are equal, then clearly $GIF_A = AIF_A = GIF_H = AIF_H$.

Problems arise however when a scatter plot is not monotone. We provide examples of different cases. Most numerical values are provided with four decimals (rounded).

Example 1. A case where $AIF_A > GIF_A$ and $AIF_H < GIF_H$

$P_1 = 10, P_2 = 20, P_3 = 40; C_1 = 20, C_2 = 50, C_3 = 60$. Then $I_1 = 2, I_2 = 5/2$ and $I_3 = 3/2$. Note that this indeed a non-monotone scatter plot.

Now, $AIF_A = 2 > 13/7 = 1.8571 = GIF_A$ while $AIF_H = 1.9149 < 2.0192 = GIF_H$.

Example 2. A case where $AIF_A = GIF_A$ and $AIF_H \neq GIF_H$

$P_1 = 10, P_2 = 20, P_3 = 30; C_1 = 20, C_2 = 50, C_3 = 60$. Then $I_1 = 2, I_2 = 5/2$ and $I_3 = 2$.
Now, $AIF_A = 13/6 = 2.1667 = GIF_A$ while $AIF_H = 2.1429 \neq 2.1154 = GIF_H$.
Example 3. A case where $AIF_A \neq GIF_A$ and $AIF_H = GIF_H$

$P_1 = 20$, $P_2 = 30$, $P_3 = 60$; $C_1 = 40$, $C_2 = 100$, $C_3 = 120$. Then $I_1 = 2$, $I_2 = 10/3$ and $I_3 = 2$.
Now, $AIF_A = 16/6 = 2.6667 \neq 26/11 = 2.3636 = GIF_A$ while $AIF_H = 30/13 = 2.3077 = GIF_H$.

Adding the data of examples 2 and 3 yields example 4, a really counterintuitive case.

Example 4. A case where $AIF_A > GIF_A$ and $AIF_H > GIF_H$

$P_1 = 30$, $P_2 = 50$, $P_3 = 90$; $C_1 = 60$, $C_2 = 150$, $C_3 = 180$. Then $I_1 = 2$, $I_2 = 3$ and $I_3 = 2$.
Now, $AIF_A = 7/3 = 2.3333 > 39/17 = 2.2941 = GIF_A$, while $AIF_H = 2.25 > 29/13 = 2.2308 = GIF_H$.

Combining Example 4 with Theorems 1 and 2 leads to the following result which is interesting for regression analysis in general.

Theorem 4

If the regression line of y over x has a positive slope, then the slope of the regression line of 1/y over 1/x is not necessarily positive too.

Conclusion

Although impact factors are well known entities and basically just simple ratios, they continue to be a source of inspiration for empirical and theoretical researchers alike. We hope that this simple contribution will prove to be useful for many colleagues. We recall that what has been established here for journal impact factors is also true for similar ratios, such as web impact factors.

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